Carnot's Theorem

State and prove Carnot's Theorem

Statement:

(I) All reversible engines working between the same temp. limits have the same efficiency.

(II) Working between the same initial and final temperature no engine can be more efficient than a reversible one.

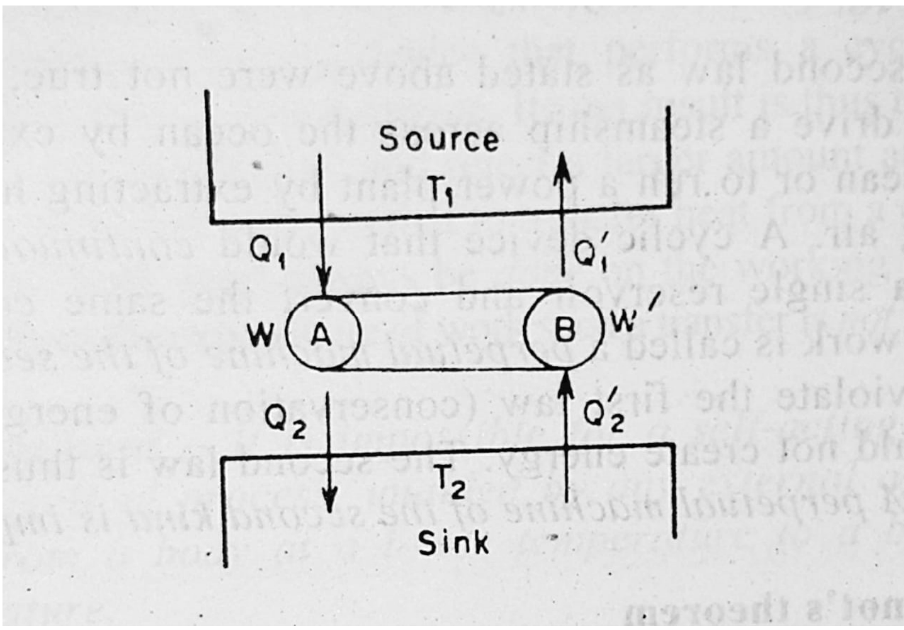

Figure-1

Proof: I) Consider two reversible engines A&B operating between the temperatures T_1 & T_2 where $T_1 > T_2$. Let the efficiency of A (η) is greater than B(η'). i.e. $\eta > \eta'$

Let A work as a heat engine & B as a refrigerator conveying heat from the stink to the source. A takes an amount of heat Q1 from the source of Temp T1 & does an external work W.

II) Consider two engines S & R of which S is irreversible & R is reversible. Both the engines are working between the same two temperature limits T1 & T2 where T1>T2. Let the efficiency η of S is greater than η' of R

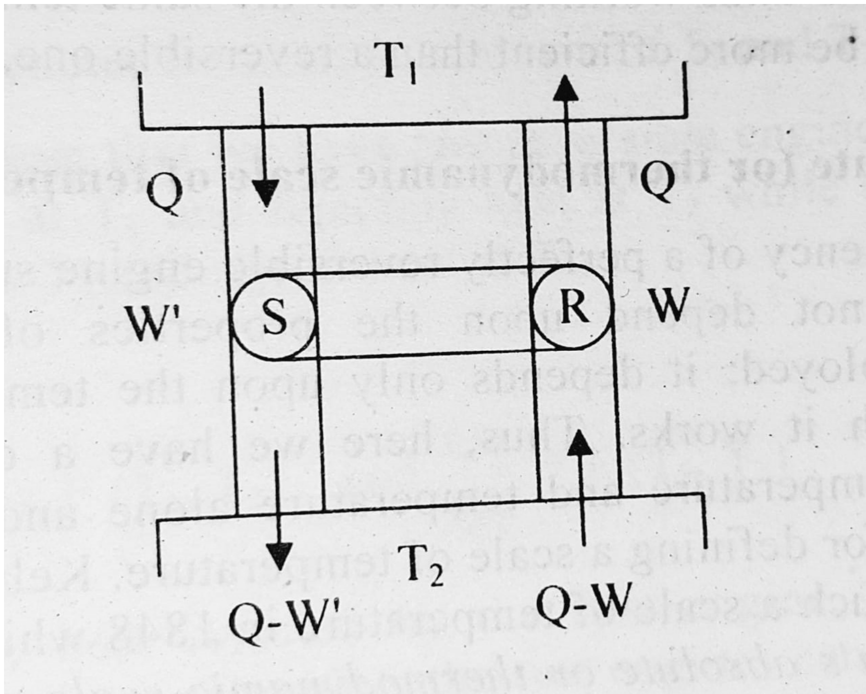

Figure -2

Let the two engines be coupled together in such a way that S drives R. So, R acts as a refrigerator. Let, S absorbs heat Q from the source of temperature T1 perform work W' and give out the rest heat Q-W' to the sink of temp T2, let R take source heat from sink & after work returns Q amount of heat to the source. So the amount of heat that R abstracts from the sink be Q – W

Now, since η > η'

Or, $\frac{W'}{Q} > \frac{W}{Q}$

Or, W' > W [-W' < -W]

Or, Q – W > Q – W'

This means R extracts more heat from the sink than S delivers to it. The net result is that the combined system extracts a certain quantity of heat (Q – W) - (Q – W') = W' > W per cycle from the sink & converts

The whole of it into work while the source remain unaffected which is the contradiction of 2nd law of thermodynamics.

Hence S cannot be more efficient than R. Thus working between the same temp limits no engine can be more efficient than a reversible one.

Describe the Carnot's cycle and Carnot engine

Carnot Cycle: A cycle in which the working substance starting from a given condition of temp, pressure & volume is made to undergo two successive expansions one isothermal & another adiabatic then two successive compressions one isothermal and another adiabatic & then brought back finally to its initial conditions is called Carnot cycle.

Carnot Engine: Sadi Carnot conceived an ideal engine to convert heat-energy to mechanical energy which is free from all imperfection of an actual engine is known as carnot engine. The different parts of carnot engine are shown below

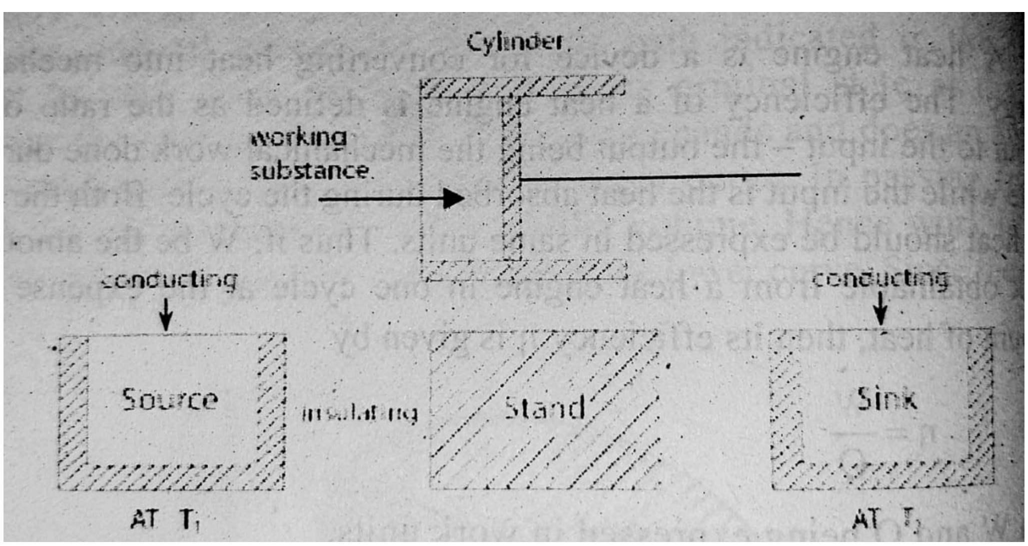

Figure-3

1) A cylinder C with perfectly non-conducting walls & a perfectly conducting bottom. It is filled with a perfectly non-conducting &

frictionless piston P. An ideal gas is enclosed in the cylinder which acts as the working substance.

2) The heat source is a hot object with very high thermal capacity is maintained at temp of T1. Any amount of heat can be taken out from it without changing its temperature.

3) The sink is a cold object with very high thermal capacity is maintained at a lower temp of T2. Any amount of heat can be rejected to it without changing its temperature.

4) A platform made up of completely insulating object on which the cylinder is placed

Obtain Expression for the work done in each operation of the carnot cycle and the work done in the cycle.

Sol^n: There are four stage of a carnot cycle. Let the engine cylinder contain m grams of air as the working substance & let its original condition be represented by the point a, let the air have pressure P1, volume v1 and the same temperature T1 K as that of the source

<u>Operation I (Isothermal expansion)</u>

Let the bottom of the cylinder be placed in contact with the source and the gas is allowed to expand slowly. The falling temp during expansion is compensated by absorption of heat from the source. So the gas expand isothermally to the point b represented by P_2, V_2 &T_1 . During the process an amount of heat energy Q_1 is absorbed from the source of constant temperature. This energy is equivalent to the amount external work w_1 done by the gas during the expansion. The amount of work done by air.

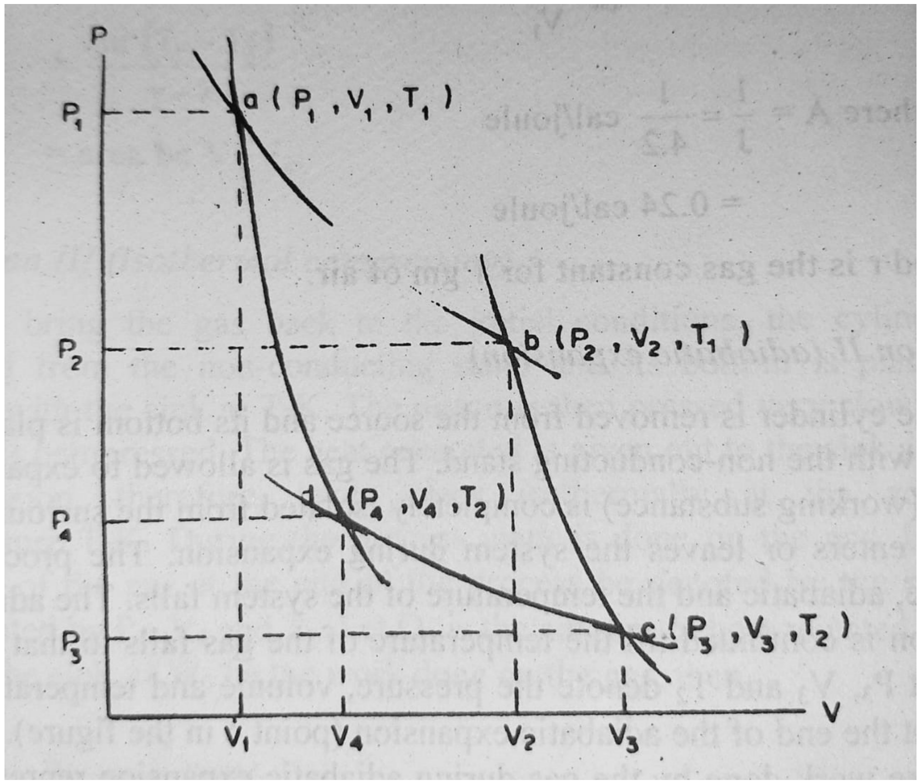

Figure-4

$W_1=\int_{v1}^{v2} PdV$

$=m\gamma T_1\int_{v1}^{v2} \frac{dV}{V}$

$= m\gamma T_1 \log\frac{dV}{V}$

$=$area V_2V_1

Heat received

$Q_1= \frac{w1}{J}$

$= \frac{m\gamma T1 \log V2/V1}{J}$

$= m\gamma T_1 A \log V_2/V_1$

Where $A=\frac{1}{J} = \frac{1}{4.2}$ cal/joule = 0.24 cal/joule

<u>Operation II (adiabatic expansion)</u>

The cylinder is removed from the source and the bottom is placed in contact with the non-conducting stand. The gas is allowed to expand. As like gas is completely isolated from the surrounding no heat enters or leaves the system during expansion.

The process is therefore adiabatic and the temp of the system falls. The adiabatic expansion is continued till the temp of the gas falls to that of the sink. Let P_3, V_3 & T_2 denote the work done by the gas during adiabatic expansion represented by bc, we have

$W_2 = \int_{v2}^{v3} PdV$

$= k \int_{v2}^{v3} \frac{dv}{v^{\gamma}}$ [for adiabatic change $PV^{\gamma}=k$]

$= k \left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{v2}^{v1}$

$= \frac{k}{\gamma-1} (V_2^{-\gamma-1} - V_3^{-\gamma+1})$

$= \frac{P_2V_2 - P_3V_3}{\gamma-1}$ [$k= P_2V_2^{\gamma}=P_3V_3^{\gamma}$]

$= \frac{m\gamma T_{1-m\gamma T_2}}{\gamma-1}$

$= \frac{m\gamma (T_{1-} \ T_{2})}{\gamma-1}$ = area bc V_3V_2

<u>Operation III (isothermal compression)</u>

To bring the gas back to the initial condition the cylinder is removed from the stand and its bottom is placed in contact with the sink at T_2k. The piston is pressed very slowly & the gas is compressed. The heat generated is given out to the sink & the compression takes place isothermally at that constant temp T_2k. During the process work is done on the gas. Let the position of the gas at the end of the process be denoted by the point d(P_4, V_4, T_3). Let Q_2 is the amount of the heat rejected to the sink. If w_3 be the work done on the gas, then

$W_3 = \int_{v3}^{v4} PdV$

$= m\gamma T_2 \log \frac{V4}{v3}$

=area c V_3V_4 d

& $Q_{2-} = \frac{m\gamma T_2 \log V_4/V_3}{J}$

= $m\gamma AT_2 \log V_4/V_3$

Operation IV (adiabatic compression)

The cylinder is then removed from the sink & its bottom is placed in contact with the isolating stand. The gas is compressed adiabatically until its temp rises to T_1k & its pressure & volume become P_1 & V_1 represented by the point a. If w_4 be the amount of work done on the gas during adiabatic compression them.

$W_4 = \int_{v_4}^{v_1} PdV$

$= \frac{-(P_4V_4-P_1V_1)}{\gamma-1}$ $\quad [=\frac{-m \ (T_2-T_1)}{\gamma-1}]$

$= \frac{(P_1V_1-P_4V_4)}{\gamma-1}$ $\quad [=\frac{-m\gamma(T_1-T_2)}{\gamma-1}]$

The net work w done by the system during the cycle is given by

$W = w_1 + w_2 - w_3 - w_4$

$= w_1 - w_3$ ($\because w_2 = w_4$)

This is represented by the area abcd.

What is efficiency of an engine? Derive an expression for the efficiency of a Carnot engine intern of the temperature of the source and the sink.

Sol^n :

The efficiency of a Carnot engine is defined as the ratio of the mechanical work done during a cycle to the heat supplied during the cycle, both the work & the heat being expressed in the same unit. It is denoted by η.

If Q_1 & Q_2 be the respective amount of heat takes in the temperature T_1k & given out at T_2k by the engine, then Q_1-Q_2 represents the amount of heat converted into work. Then,

$$\eta = \frac{work\ done}{hea\ \ drawn\ from\ the\ source}$$

$$= \frac{heat\ converted\ into\ work}{heat\ drawn\ from\ the\ source}$$

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Efficiency in term of the temperature of the source & sink.

From the expression for the work done during each operation of a Carnot cycle we get,

$$\frac{Q_1}{Q_2} = \frac{W_1}{W_3} = \frac{mrA\ T_1 \log\left({}^{v_2}/_{v_1}\right)}{mrA\ T_2 \log\left({}^{v_3}/_{v_4}\right)}$$

Now, for isothermal process lies on ab & cd. So for an ideal gas, $p_1v_1 = p_2v_2$ & $p_3v_3 = p_4v_4$

Again, for adiabatic process lies on bc & ad. So for an ideal gas, $p_2v_2^{\gamma} = p_3v_3^{\gamma}$ & $p_4v_4^{\gamma} = p_1v_1^{\gamma}$

Multiplying these four equations we have ,

$$p_1v_1p_3v_3p_2v_2^{\gamma}p_4v_4^{\gamma} = p_2v_2p_4v_4p_3v_3^{\gamma}p_1v_1^{\gamma}$$

- $v_1v_2^{\gamma}v_3v_{4\gamma} = v_2v_3^{\gamma}v_4v_1^{\gamma}$
- $\frac{v_2^{\gamma}v_4^{\gamma}}{v_2v_4} = \frac{v_3v_1^{\gamma}}{v_3v_1}$
- $(v_2v_4)^{\gamma-1} = (v_3v_1)^{\gamma-1}$
- $v_2v_4 = v_3v_1$
- $\frac{v_2}{v_1} = \frac{v_3}{v_4}$

Hence, $\frac{Q_1}{Q_2} = \frac{T_1 \log\left({}^{v_2}/_{v_1}\right)}{T_2 \log\left({}^{v_3}/_{v_4}\right)}$

$$= \frac{T_1 \log (^{v_2}/_{v_1})}{T_2 \log (^{v_2}/_{v_1})} = \frac{T_1}{T_2}$$

➤ $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

So, the efficiency $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$

EX: An engine absorbs 2.56×10^6J heat energy from a source at 600K temperature & reject 5.12×10^5J heat energy to a sink at lower temp. Find the temp of the sink & efficiency of the engine.

Sol^n: Here,

$Q_1 = 2.56\times 10^6$J

$Q_2 = 5.12\times 10^5$J

Higher temp $T_1 = 600$K

Lower temp $T_2 = ?$

Efficiency $\eta = ?$

We know,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

➤ $T_2 = \frac{Q_2}{Q_1} \times T$

$$= \frac{5.12\times 10^5 J}{2.56\times 10^6 J} \times 600K$$

$$= 120k$$

Again , $\eta = \frac{T_1 - T_2}{T_1} = \frac{(600-120)K}{600K} = 0.80 = 80\%$

Ans: 120K , 80%

EX: The efficiency of a Carnot engine working between temp 800K & 400K is equal to that working between temp T K & 900K. Find the temp T.

Sol^n: Here,

In the 1st case, Higher temp T_1= 800K

Lower temp T_2= 400K

Efficiency η= ?

We know, $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400K}{800K}$ = 0.5= 50%

In the 2nd case, Higher temp T_1=T=?

Lower temp T_2=900K

Again, $\eta = 1 - \frac{T_2}{T_1}$

- ➢ $0.5 = 1 - \frac{900K}{T_1}$
- ➢ $\frac{900K}{T_1} = 0.5$
- ➢ T_1 = 1800K

Ans : 1800K

Q: What is refrigerator? Describe the construction & working principle of a refrigerator.

Refrigerator: Refrigerator is a device in which heat energy is brought out such a way that the temperature inside it becomes much lower than the room temp. As a result, our preserve able food like fish, meat, uncooked and cooked curry remain fresh for long time.

Construction: The main part of a refrigerator is a rectangular cooling chamber which is surrounded by a heat conducting coil of metal pipe. One end of the coil is connected with the is let of a compressor kept at the lower part of the refrigerator. Other end of the coil is connected to another coil with a valve. Another condensation coil is mounted on a fins at the back of the refrigerator connected with compressor.

A special type of refrigerant is flown through the coils with high pressure & it absorbs heat from the environment & prickly connected into vapor. The compressor is running by an electric motor.

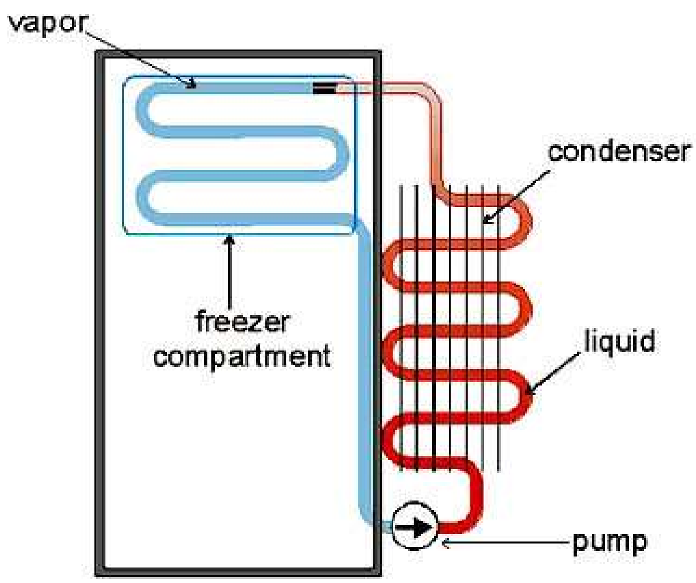

Figure-6

<u>Working principle:</u> When a compressor is started the pressure of the vaporing coil is reduced & the refrigerant is quickly vaporized. For this latent heat is required. The refrigerant is converted into vapor absorbing latent heat from the cooling chamber. As a result, low temp is crested inside the cooling chamber. The pump compressed the refrigerant gas at high pressure & flows through the condensation coil. This gas release heat at the atmosphere & becomes liquid. This liquid is again flows through the vaporing coil. This cycle of condensation & evaporation continues & low temp in the cooling chamber exists. If cooling chamber becomes colder than requirement, then the power supply is cut off. There is a thermostat inside the $R^{\gamma.}$ When the temp becomes lower than requirement the thermostat automatically start functioning.