Physics I

Entropy

The property of the reversible cycle expresses by the equation

$$\oint \frac{dQ}{T} = 0$$

enable us to define a new function of sate called Entropy of the system.

The change in entropy of a system is defined when it is changed from state 1 to state 2 is

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} .$$

Properties:

1. Net change in entropy in a reversible cycle is zero.
2. Entropy increases during irreversible operations.
3. Entropy remains constant for all adiabatic frictionless processes.
4. Entropy is always on the increase.
5. Entropy is a measure of the disorder in molecular arrangement of a substance.

Example: 21.2: Find the change in Entropy when 1 gm of ice at 0^o C changes to water at 0^o C under 1 atm pressure. Latent heat of the fusion of ice = 80 cal/gm

Solution: Change in Entropy

$$S_2 - S_1 = \frac{Q}{T}$$

$$= \frac{80}{273}\ cal/k$$

Here,

$$Q = 80\ cals$$

$$T = (0 + 273)K = 273\ k$$

Example: 21.3: Find the difference in entropy between 1 gm of water at 100^o C and 1 gm of steam at 100^o C at atm. (Latent heat of steam at $100°C = 540 cals/gm$)

Solution: We know,

Change in Entropy

$$S_2 - S_1 = \frac{Q}{T}$$

$$= \frac{540}{373}\ cal/k$$

$$= 1.448\ cal/k$$

Here,

$$Q = 540\ cal$$

$$T = (100 + 273)K = 273k$$

Example: 21.4: Calculate the increase in Entropy when 1gm of ice at -10^o C is converted into steam at 100^o C.

Solution:

i. Increase in Entropy when the temperature of 1 gm of ice increase from -10^o C to 0^o C

$$dS = \int_{T_1}^{T_2} \frac{dQ}{T}$$
$$= \int_{T_1}^{T_2} \frac{msdT}{T}$$
$$= ms[\ln T]_{T_1}^{T_2}$$
$$= ms \ln \frac{T_2}{T_1} = 1 \times 0.5 \ln \frac{273}{263}\ K$$
$$= 0.01865$$

Here,

$m = 1gm$

$Specific\ heat\ of\ ice\ S = 0.5$

$T_1 = (-10 + 273) = 263\ K$

$T_2 = (0 + 273) = 273\ K$

ii. Increase in Entropy when 1 gm of ice at 0^o C is converted into water at 0^o C

$$dS = \frac{Q}{T} = \frac{80}{273} = 0.293\ cal/k$$

iii. Increase in Entropy when the time of 1 gm of water is raised from 0^o C to 100^o C

Here,

$m = 1$

$S = 1$

$T_1 = (0 + 273) = 273\ K$

$T_2 = (273 + 100) = 373$

$$dS = \int_{T_1}^{T_2} \frac{dQ}{T} = 0.312\ cal/K$$

iv Increase in Entropy when 1 gm of water at 100^o C is converted into steam at 100^o C

$$dS = \frac{Q}{T}$$
$$= \frac{540}{373}$$
$$= 1.44\ cal/K$$

So total increase in Entropy $= 0.01865 + 0.293 + 0.312 + 1.448 = 2.07165\ cal/K$

Show that the entropy for reversible process remain constant and increase for irreversible process

In reversible cycle say in a Carnot cycle there are two isothermal and two adiabatic process. Total change in entropy for a whole reversible process

$$\int dS = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

Again, for a reversible cycle $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

Hence,

$$\int dS = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

So, $S = constant$

Therefore, the entropy for reversible process remain constant.

The entropy of a system in irreversible process

Let, two objects with T_1 & T_2 temperature are completely isolated from the environment are in contact with each other. If $T_1 > T_2$ then let dQ amount of heat is transferred from warm object to cold object, the warm object losses dQ amount of heat and cold object gains dQ amount of heat.

Therefore $-\frac{dQ}{T_1} = Decrease\ of\ entropy\ of\ the\ warm\ object.$

$$\& \ \frac{dQ}{T_2} = Increase\ of\ entropy\ of\ the\ cold\ object.$$

Therefore the change in entropy of the system, $dS = -\frac{dQ}{T_1} + \frac{dQ}{T_2}$

Since, $T_1 > T_2$, so $dS > 0$ that is the change of entropy is always positive.