PHYSICS I

WAVE MOTION

Wave motion: Wave motion is the transport of energy and momentum from one point in space to another without the transport of matter.

Types: 1. Mechanical wave

2. Electromagnetic wave.

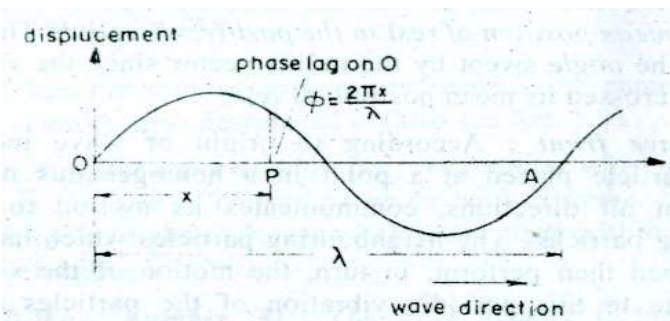
Media is necessary for mechanical wave motion (sound, water wave) whereas no media is necessary for electromagnetic wave motion (light, radio).

There are two types of wave motion:

1. Transverse WM
2. Longitudinal WM

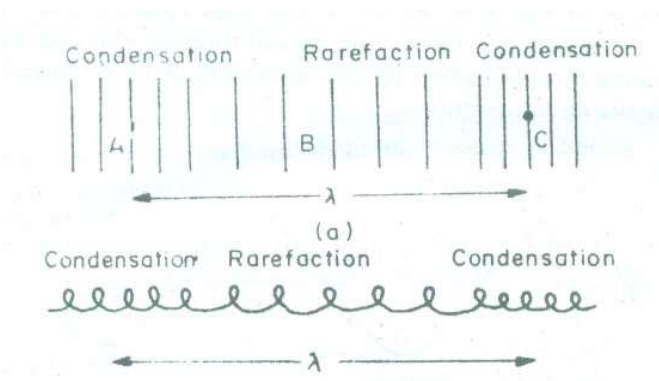
1. Transverse Wave Motion: In this motion the particles of the medium oscillate up and down about their mean & equilibrium position in a direction right angles to the direction of propagation of wave motion.

Ex: wave travelling along a stretched string, water liquids etc.



2. Longitudinal Wave Motion: Here the particles of the medium oscillate to and for about their mean or equilibrium position along the direction of propagation of the wave motion.

Ex: Wave produced in a spring or helix when one end of it is suddenly compressed or pulled out & then released or sound waves in air.



Wave length: The distance travelled by the wave in the time in which the particle completes one vibration.

Amplitude: Amplitude is the maximum displacement of the particle.

Time Period(T): It is the time to complete one vibration.

Frequency(n): The number of complete oscillations made by a particle of the medium.

Let,

Frequency = n

Time taken to complete n vibrations = 1 sec.

Time taken to complete 1 vibration T = 1/n sec.

By Definition,

$$T = \frac{1}{n}$$

or , nT=1

or, Frequency × Time period =1

<u>**Angular Frequency**</u> **(ω):** The rate of change of phase with time is called angular frequency. Since in one complete cycle phase change of 2π occurs in a time T,

Angular frequency

$$\omega = \frac{2\pi}{T}$$

$$= 2\pi n. \quad \text{[unit: rad/sec.]}$$

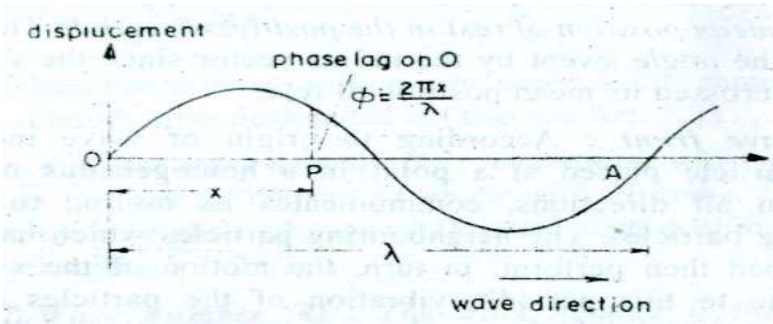
Velocity: The distance traveled by the wave in time period T,

$$\text{Velocity } V = \frac{wave\ leng}{time\ period} = \frac{\lambda}{T}$$

$$\Rightarrow V = n\lambda \qquad [\, n = 1/T]$$

Expression for the plane progressive wave: A plane progressive wave is one which travels onward through the medium in a given direction without attenuation.

Consider a wave started at the point 0 and moves along x-axis as shown in figure.



The equation of the motion of the particle:

$$y = a\sin\omega t \qquad \textbf{(1)}$$

Where y is the displacement of the particle at time t, a is its amplitude, ω is it's angular velocity.

For a particle at P at a distance x from 0, This phase difference is φ.

The equation of the particle at P is,

$$y = a\sin(\omega t - \phi) \qquad \textbf{(2)}$$

For a path difference λ the phase difference is 2π.

For a distance x the phase difference is $\varphi = \frac{2\pi}{\lambda}x$

Now from equation (2) we have,

$$y = a\sin(\omega t - \frac{2\pi}{\lambda}x)$$

$$= a\sin(\omega t - kx) \qquad \textbf{(3)}$$

[k = 2π/λ is the angular wave no.]

We know,

ω = 2π/T

= 2πn

= 2πV/λ

Here,

V = nλ

=> n = V/λ

From equation (3)

$$y = a \sin \left(2\pi \frac{V}{\lambda} t - \frac{2\pi}{\lambda} x\right)$$

$$= a \sin \frac{2\pi}{\lambda} (vt - x)$$

$$y = a \sin k (vt - x)$$

Again,

$$y = a \sin \frac{2\pi V}{\lambda}\left(t - \frac{x}{V}\right) \qquad [\, n = 1/T]$$

$$= a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad [\, v/\lambda = n = 1/T]$$

If the wave travels towards the left, x becomes negative & we have

$$y = a \sin \frac{2\pi}{\lambda} (Vt + x)$$

Ex. 4.1: The displacement of a particle executing SHM at any instant of time is given by,

$$y = 0.1 \sin 2\pi\,(340\,t - 0.15)$$

Calculate the amplitude, wave velocity, wave length, frequency, & time period.

Solution:

The general equation of a SHM is given by,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \qquad \textbf{(1)}$$

Here,

$$y = 0.1 \sin 2\pi\,(340t - 0.15) \qquad \textbf{(2)}$$

Comparing equation (1) & (2)

Amplitude, a = 0.1 meter

Wave length, λ = 1 meter

Wave velocity, V = 340 m/s

Frequency, $n = V/\lambda = 340/1 = 340$ Hz

Time period, $T = 1/n = \frac{1}{340}$ sec

Ex. 4.2: A simple harmonic wave of amplitude 8 units traverse a line of particles in the direction of positive x-axis. At any given instant of time the displacement of a particle at a distance of 10 cm from the origin is +6 units while that for a particle at a distance of 25 cm from the origin is +4 units. Calculate the wavelength.

The equation of a simple harmonic wave can be written as

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$\text{or, } y/a = \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

In the first case,

y = +6, a = 8 and x = 10cms

$$6/8 = \sin 2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) \qquad (1)$$

In the 2nd case,

y = +4, a = 8 and x = 25cms

$$4/8 = \sin 2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) \qquad (2)$$

From equation (1) we have

$0.75 = \sin 2\pi \left(\frac{t}{T} - \frac{10}{\lambda} \right)$

But $\sin\left(\frac{48.6\pi}{180}\right) = 0.75$

So, $2\pi \left(\frac{t}{T} - \frac{10}{\lambda} \right) = \left(\frac{48.6\pi}{180}\right)$

or, $\frac{t}{T} - \frac{10}{\lambda} = \frac{48.6}{360}$ (3)

From equation (2) we have

$0.5 = \sin 2\pi \left(\frac{t}{T} - \frac{25}{\lambda} \right)$

But $\sin\left(\frac{\pi}{6}\right) = 0.5$

So, $4/8 = 2\pi \left(\frac{t}{T} - \frac{25}{\lambda} \right) = \frac{\pi}{6}$

or, $\frac{t}{T} - \frac{25}{\lambda} = \frac{1}{12}$ (4)

Subtracting (4) from (3) we have

$\frac{25}{\lambda} - \frac{10}{\lambda} = \frac{48.6}{360} - \frac{1}{12}$

or, $\lambda = 290.8\ cm.$

Ex.4.3: The velocity of a simple harmonic wave is 30 cm/s. At a time, t = 0 the displacement of a particle is given by,

$$y = 4 \sin 2\pi \left(\frac{x}{100}\right)$$

Find the equation for the displacement at a time t = 2 sec.

Solution:

The general equation of a SHM is

$$y = a \sin \frac{2\pi}{\lambda} (vt - x)$$

At t = 0 ,

$$y = a \sin \left(\frac{-2\pi x}{\lambda}\right)$$

$$= - a \sin \left(\frac{2\pi x}{\lambda}\right) \quad \textbf{(1)}$$

At $t = 0$, the given equation is

$$y = 4 \sin 2\pi \left(\frac{x}{100}\right) \qquad \textbf{(2)}$$

Comparing equation (1) & (2)

$a = -4, \quad \lambda = 100$ cm

At $t = 2$ sec,

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -4 \sin \frac{2\pi}{100}(30 \times 2 - x)$$

$$y = 4 \sin\left\{2\pi\left(\frac{x}{100}\right) - \frac{6\pi}{5}\right\}$$

Here,

$\lambda = 100$cm

$a = -4$

$v = 30$ cm/s

which is the displacement at a time $t = 2$ sec.

Ex. 4.5 A plane progressive wave train of frequency 400 cycles per second has a phase velocity of 480m/sec. (i) How far apart are two points 30degree out of phase? (ii) What is the phase difference between two displacements at a given point at times 10^{-3} sec apart?

Solution

The equation of a plane progressive wave is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

where $\frac{2\pi}{\lambda}$(vt - x) is the phase angle of a point at a distance x from the origin at time t.

so, the phase angle of a point at a distance x_1 from the origin at time t = $\frac{2\pi}{\lambda}(vt - x_1)$ and

the phase angle of a point at a distance x_2 from the origin at time t = $\frac{2\pi}{\lambda}(vt - x_2)$

Hence the path difference between the two points

$$\frac{2\pi}{\lambda}(vt - x_1) - \frac{2\pi}{\lambda}(vt - x_2)$$

$$= \frac{2\pi}{\lambda}(x_2 - x_1)$$

$$= \frac{2\pi v}{\lambda}(x_2 - x_1)/v$$

$$= 2\pi n(x_2 - x_1)/v$$

Since, $\frac{v}{\lambda} = n$, the frequency of the wave.

The path difference between the two points

30 degree = $\frac{30\pi}{180}$ rad = $\frac{\pi}{6}\ rad$

So, $2\pi n(x_2 - x_1)$ /v = $\frac{\pi}{6}$

Or, $2n(x_2 - x_1)$ /v = $\frac{1}{6}$

Here, n = 400, v = 480m/s = 480×10^2 cm /sec

So, $2 \times 400 \times(x_2 - x_1)$ /(480×10^2)= $\frac{1}{6}$

Or, $x_2 - x_1 = \frac{(480\times10^2)}{2\times400\times6}$ = 10 cm = 0.1 m.

(ii) Again the phase angle at a point x from the origin at time t_1

$$= \frac{2\pi}{\lambda}(vt_1\text{-}x)$$

And the phase angle at a point x from the origin at time t_2

$$= \frac{2\pi}{\lambda}(vt_2\text{-}x)$$

The phase difference between the two points

$$= \frac{2\pi}{\lambda}(vt_2 - x) - \frac{2\pi}{\lambda}(vt_1 - x)$$

$$= \frac{2\pi v}{\lambda}(t_2\text{-}t_1)$$

$$= 2\pi n\ (t_2 - t_1)$$

So, the phase difference between the two points

$$= 2 \times \pi \times 400 \times 10^{-3} = 0.8\,\pi\ rad = 144\ degree.$$